# Nonlinear slight parameter changes detection: a forecasting approach

Jeremias Sulam[§], Gastón Schlotthauer[†§], and María E. Torres[†§]

[§]Laboratorio de Señales y Dinámicas no Lineales,
Facultad de Ingeniería - Universidad Nacional de Entre Ríos,
[†]CONICET
Argentina
metorres@santafe-conicet.gov.ar

**Abstract.** In many biological systems it is crucial to detect changes, as accurate as possible, in the parameters that govern their dynamics. In this work we propose a new method to perform an online automatic detection of such changes, making use of a well known nonlinear forecasting algorithm. The approach takes advantage of the characterization of an interval of a signal by the reconstruction of its phase space through time-delay embedding. To this end, the optimal delay and embedding dimension are estimated, and a method is proposed for determining the forecasting parameters, after which it is possible to predict future values of the studied signal. In this novel approach the method is used as a way of detecting changes in the dynamics of a system, given that the forecast is performed using a template of the signal where its parameters remain constant. At this point, the measure of the prediction error is used to detect a change in the dynamics of the system. We also propose a second estimator of this change, namely *prediction failure*, which is a stronger binary estimator of change in the dynamics. The results are analyzed by a cumulative sum algorithm ($CUSUM$) to obtain a detection point. In order to test their behavior, both methods are applied to deterministic discrete and continuos synthesized data, and to a simulated biological model.

**Keywords:** nonlinear event detection, nonlinear forecasting, prediction error.

## 1 Introduction

Frequently, changes in the dynamics of a system are due to the change of just one of its parameters. In particular, in biological systems their governing laws might present strong nonlinearities. Their behavior can be chaotic, and these changes can be related to a pathological condition of the biological system. Therefore, the correct analysis of the outcome signal of such a system is crucial to the understanding and detection of disorders.

When nonlinear dynamics are present, the conventional analysis based on Fourier theory sometimes yields results that can not represent the underlying dynamics in its full magnitude. A variety of methods have been proposed to undertake the study of such signals (correlation dimension, Lyapunov exponents and other complexity measures), and they have been applied in several fields, including the analysis of biological systems and signals.

When the dynamics can be represented by a set of differential equations, analytical methods can provide information to characterize the corresponding systems. However, in real applications this representation is not always available and the system's reconstructed phase space by means of time-delay embedding becomes an important tool of analysis. It enables the study of the dynamics of a systems from one of its output signals. These techniques have been widely applied to the study of changes and non-stationarity in biological systems [1–4].

In this paper we propose a novel approach to this aim by using the prediction error of a nonlinear forecasting algorithm as an indicator of slight parameters changes. Previous works were focused on the use of forecasting algorithms to either detect changes in a system or to prove chaotic behavior. Altunay et al. [5] studied the detection of epileptic seizure from electroencephalogram recordings by means of a linear prediction method. Sugihara et al. [6] studied a nonlinear short term forecasting algorithm aiming to detect chaos from measurement noise. The authors state that under chaotic dynamics the accuracy of the nonlinear predictor decreases with the increasing prediction time interval, at a rate which gives an estimate of the maximum Lyapunov exponent, while uncorrelated noise should not depend on the prediction interval. A nonlinear prediction algorithm was used by Dushanova et al. [7] to detect the presence of chaos in a signal, and was applied to the study of single-trial *readiness potentials*. In their method the authors use the same signal under analysis for both library and prediction, and their method provides good results for chaotic sequences in between non-chaotic deterministic signals. However, the approach doesn't prove to be effective in detecting small changes in the parameter of system under chaotic behavior, and the detection relies on the overall change in the dynamics, not necessarily on the presence of chaos, as stated by the authors.

In 1997, Torres et al. [8] proposed a method to detect slight parameter changes in nonlinear systems. In 2003, Torres et al. [9] proposed an automatic method to perform this task. The authors applied this approach to the automatic detection of seizure episodes of petit-mal epilepsy [10]. Using multiresolution entropies (MRE, a combination of wavelet decomposition and entropies evaluated on sliding windows by scales), they showed that the changes could be found as statistical variations at each scale of analysis. After extracting the corresponding principal component of the MRE matrix, the point in which the complexity of the system changed was obtained using a statistical change detection algorithm.

In this work, we present a novel approach to the automatic detection of slight changes in a system. Given a signal which is considered to be the output of an unknown system, the proposed method allows to detect, if present, changes in the parameters of the system, making use of a nonlinear non-parametric prediction algorithm. The method is explained and applied to deterministic maps and continuos flow systems, as well as to a simulated biological data.

# 2 Methods

## 2.1 Phase space embedding

Given a deterministic system, its state at a time $n$ can be specified by a vector $\mathbf{y}_n \in \mathbb{R}^m$ . The evolution of the states determines trajectories in a phase space. In the particular case of nonlinear chaotic systems, they present complicated geometrical shapes, often called *strange attractors*. However, in real cases we don't have access to either the states of the system nor to its governing equations, but to a series of scalar measures. Then, it is necessary to reconstruct the evolution of states of the system.

Given a time series with real samples $x_n$, a delay reconstruction in $m$ dimensions is obtained by the vectors $\mathbf{y}_n$ [11], given by:

$$\mathbf{y}_n = \left(x_{n-(m-1)\tau}, x_{n-(m-2)\tau}, ..., x_{n-\tau}, x_n\right) \tag{1}$$

where $m$ is the embedding dimension (minimal number of independent variables needed to represent the dynamics of the system), and $\tau$ is the reconstruction delay. The trajectories provided by the reconstructed vectors $\mathbf{y}_n$ will approximate the original trajectories of the real states of the system as long as $m$ is larger than twice the *box counting dimension* of the attractor [12]. It is easy to determine the embedding parameters when some information about the system, such us the number of independent variables, is available. However, when the system is unknown, it is necessary to estimate the optimal values. In most practical applications, the product $m\tau$ is more important than the individual values, since $m\tau$ is the time span represented by each state vector $\mathbf{y}_n$.

Several methods have been proposed to estimate $\tau$. In this work we set the delay time to be the first minimum of the mutual information of the signal, as proposed by Fraser et al. [13].

## 2.2 Nonlinear forecasting

In this novel method, given a signal we select a *template* from it, in a time interval where the parameters of the corresponding systems are assumed to remain constant. This template will serve as a library used to predict future samples of the signal.

Let $x_n$, $n = 1...N$ be the sampled signal. The template, of length $L$, will then be chosen to be $x_n^t = x_n$, $n = l...l + L - 1$. This length will depend on the system under study. We then perform the phase state reconstruction of the template through time delay embedding, evaluating for that purpose the optimal embedding dimension $m$ and time delay $\tau$. The dynamics of the system can either be periodic, quasiperiodic or chaotic.

Then, under the hypothesis that the studied signal is the outcome of some deterministic system, it is possible to predict future values of the time series making use of the information of the trajectories of the phase space. We apply here the algorithm proposed by Schreiber [11], so we consider a neighborhood in the reconstructed phase space, centered at $\mathbf{y}_n$ and of radius $\epsilon$, namely $\mathscr{U}_n$. Then we find the average of the future samples corresponding to every state included in the neighborhood:

$$\hat{x}_{n+\Delta n} = \frac{1}{|\mathscr{U}_n|} \sum_{\mathbf{y}_k \in \mathscr{U}_n} x^t_{k+\Delta n}, \tag{2}$$

where $\hat{x}_{n+\Delta n}$ stands for the prediction $\Delta n$ samples into the future, $|\mathscr{U}_n|$ denotes the number of elements $\mathbf{y}_k$ found inside $\mathscr{U}_n$, and $x^t_{k+\Delta n}$ is the value $\Delta n$ samples ahead of the one corresponding to the state $\mathbf{y}_k$ reconstructed from the template. For this algorithm we need to determine the scalar radius ($\epsilon$) and how far into the future the forecast can be made ($\Delta n$). There are basically two ways of defining $\epsilon$: either choosing a fixed value for the parameter, or determining a number of neighbours to be included in $\mathscr{U}_n$, adapting the radius to each $\mathbf{y}_n$. In this work we will consider a fixed radius $\epsilon$, so we express it in terms of the minimum distance between neighbors in the template reconstructed phase space ($r_{min}$):

$$r_{min} = \max_i \left\{ \min_{j \neq i} \left\{ \left\| \mathbf{y}_i - \mathbf{y}_j \right\| \right\} \right\}, \tag{3}$$

where $||\cdot||$ represents here the Euclidean distance. Setting $\epsilon = r_{min}$ ensures that at least one neighbor will be found for each state in the template. The prediction time is set $\Delta n = 1$.

This is a non parametric algorithm where the predicted samples rely on the characteristics of the system expressed by the trajectories of the reconstructed attractor, for which it is necessary to have a set of samples from which to make the predictions. The efficiency of the method depends on the value of $\epsilon$, since a very small radius would not allow to find neighbors and hence no prediction could be obtained. Otherwise, if the radius is too large, the accuracy of the predicted sample would be very poor since many different trajectories could be taken into account and averaged out.

### 2.3 Dynamics change indices

In order to perform the detection of a change in the parameters of the system we propose two indices. Assuming that the real future samples are known, the absolute error of the forecasting method can be quantified as:

$$e_{n+\Delta n} = |x_{n+\Delta n} - \hat{x}_{n+\Delta n}|, \tag{4}$$

where $|\cdot|$ stands for the absolute value. If we compute the prediction along the whole signal, $e_n$ will therefore be a measure of the likelihood of the actual current dynamics of the system to that of the template.

We introduce here a second index for the dynamical change detection, which we name *prediction failure* $E_n$. It is defined by

$$E_n = \begin{cases} 0 & \text{if } |\mathscr{U}_n| \neq 0 \\ 1 & \text{if } |\mathscr{U}_n| = 0 \end{cases} \tag{5}$$

Therefore, if at least one neighbor is found inside the hypersphere of radius $\epsilon$ centered at $\mathbf{y}_n$, $E_n$ will be zero. Otherwise, if no neighbors are found in $\mathscr{U}_n$, the algorithm will not be able to perform any forecast and so $E_n = 1$. Thus, the prediction failure is a binary measure of how good the template is to predict the future samples of the given time series $x_n$.

This suggests that if the predictions are performed with a template taken from an interval where the parameters of the underlying system are assumed to be constant, both the prediction error and the prediction failure would increase their mean values as the dynamics of the system changes. Indeed, a change in some parameter of the system will be reflected on the shape of the reconstructed attractor. So, when searching for neighbors in the trajectory after the change, either the states inside $\mathscr{U}_n$ will cast bad predictions, or no neighbor will be found at all.

In order to perform this detection automatically, we implemented and applied a cumulative sum (CUSUM) algorithm to the absolute error prediction, enabling to obtain a detection point given an appropriate threshold. The CUSUM algorithm was first proposed by Page [14]. The method consists in studying the logarithmic likelihood rate, $S_k$, which presents a negative slope as long as the mean value doesn't change, and a positive slope after the change. The stopping point is the sample at which the difference between $S_k$ and its minimum value $m_k$ exceeds certain threshold, $h_k$, which is a free parameter [15].

## 3 Results and Discussion

In order to test the method described in the previous section, we have applied it to three simulated data with well known chaotic behavior, corresponding to discrete maps and continuos systems. We have imposed slight changes in only one parameter governing their complex dynamics.

### 3.1 Henon Map

The Henon map is a discrete-time system given by:

$$\begin{cases} x_{n+1} = 1 - \alpha\, {x_n}^2 + y_n \\ y_{n+1} = \beta\, x_n \end{cases} \tag{6}$$

For certain values of the parameters $\alpha$ and $\beta$ the system's dynamics is chaotic [12]. In this example, the signal was obtained using the classical values $\alpha = 1.40$ and $\beta = 0.30$, except from $n = 2001$ to $n = 3000$, where $\alpha = 1.32$. Note that there are two abrupt changes in the value of $\alpha$. The resulting data is shown in Figure 1.a, where the vertical lines indicate where the parameter $\alpha$ changes.

In this example, the embedding parameters are $\tau = 1$ and $m = 2$. We used as prediction parameters $\Delta n = 1$ and $\epsilon = 0.3\, r_{min}$. The template was taken as the first 1000 samples of the signal. The obtained prediction error $e_n$ is shown in Figure 1.b, and the prediction failure index $E_n$ in Figure 1.c. A noticeable increase in the mean value of $e_n$ can be observed in all the interval where the

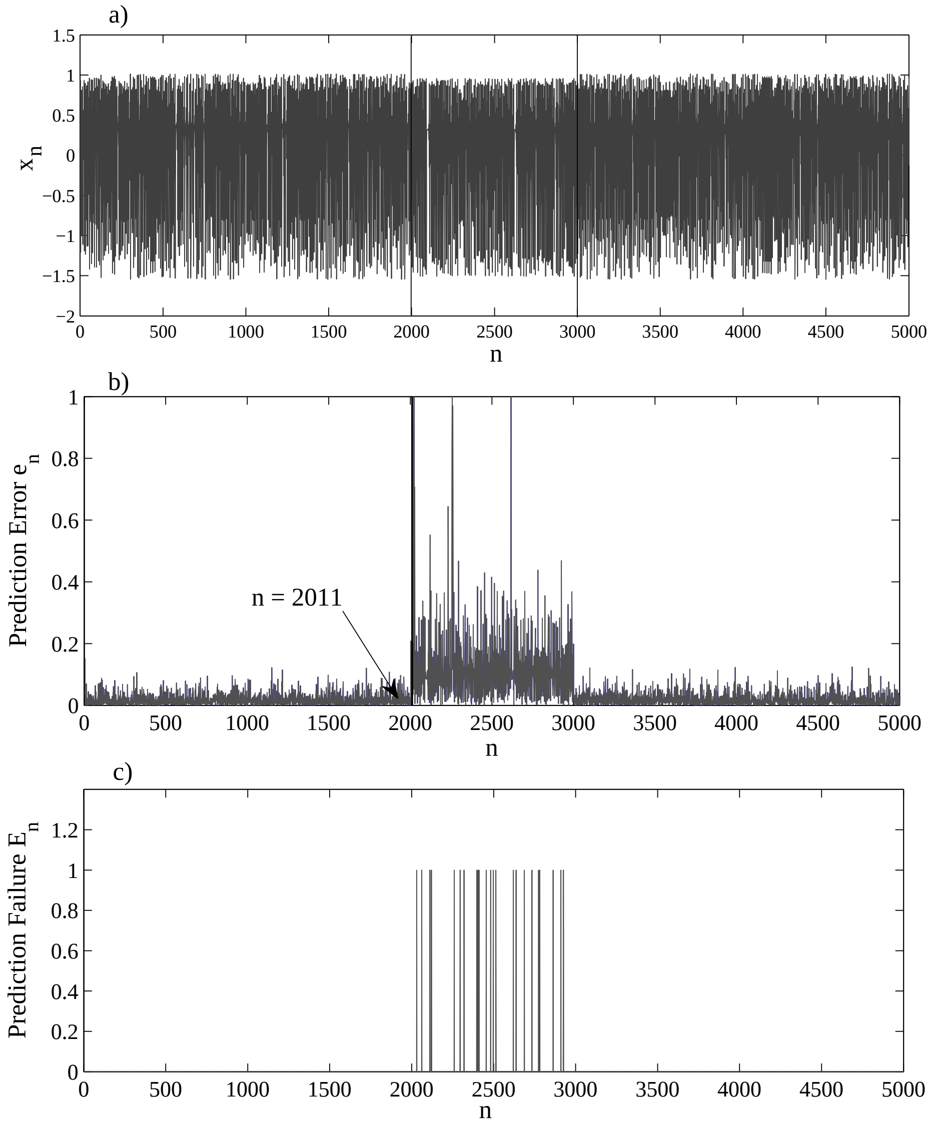


**Fig. 1.** Henon map. *a)* Henon-map signal studied, where the parameter is changed in the interval 2001–3000. *b)* Prediction error $e_n$ of the signal in (a). The detection point $n = 2011$ has been obtained by the CUSUM analysis. *c)* Prediction failure $E_n$.

parameter differs from the one in the template. The prediction failure index intermittently goes to one as the change in the dynamics is such that the method can't find any neighbors, providing strong evidence of change in the system.

The value of $\epsilon$ accounts for the sensitivity of the algorithm. If the reconstructed phase space of the template (or attractor in the case of chaotic dynamics) is not

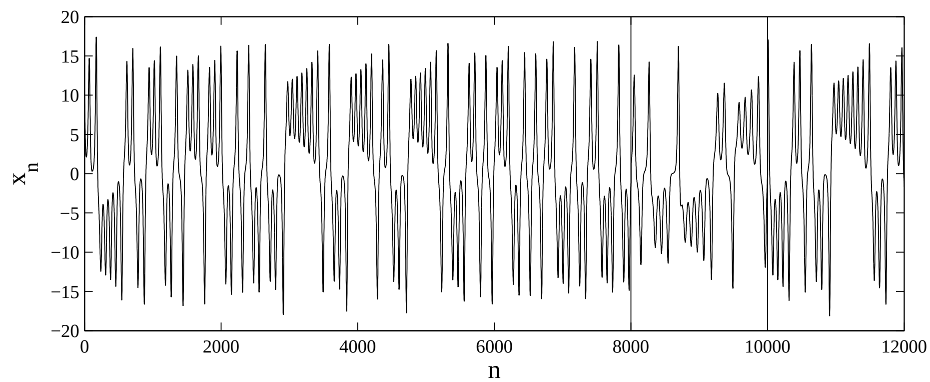


**Fig. 2.** Signal $x(t)$ corresponding to Lorenz system, with a change of parameter $\beta$ in between samples 8000 and 10000.

too disperse (i.e. the trajectories don't diverge too quickly) then the radius might be set to a lower value than $r_{min}$ and still have neighbors for every state of the template. Consequently, the algorithm would be more specific and more subtle changes might be detected.

On the other hand, the bigger the template is, the more number of trajectories will be reconstructed in the attractor of the system. Therefore, the distance $r_{min}$ will be lower, which means that smaller radii could be used in the algorithm, making it more specific and sensitive.

### 3.2 Lorenz System

The Lorenz system is a well know continuos system given by three ordinary differential equations, originally developed as a model for convection between plates. It is given by:

$$\begin{cases} x'(t) = \sigma(y - x) \\ y'(t) = x(\rho - z) - y \\ z'(t) = xy - \beta z \end{cases} \tag{7}$$

Its chaotic behavior for some values of its parameters has became a symbol of chaos theory, and Lorenz system has became one of the most studied ones.

In this example we take as the signal under study the first component $x(t)$ of the system in (7). Classical parameters $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$ have been considered, with a sampling frequency of 100 samples per time unit. The Runge-Kutta method was employed. A 2000 samples segment corresponding to $\beta = 1.5$ was added in between the signal, from sample 8000 to 10000. The resulting signal is shown in Figure 2.

The proposed algorithm was then run using $\epsilon = r_{min}$, with embedding parameters $\tau = 12$ and $m = 3$. The first 3000 samples of the signal have been selected as the template for the phase space reconstruction.

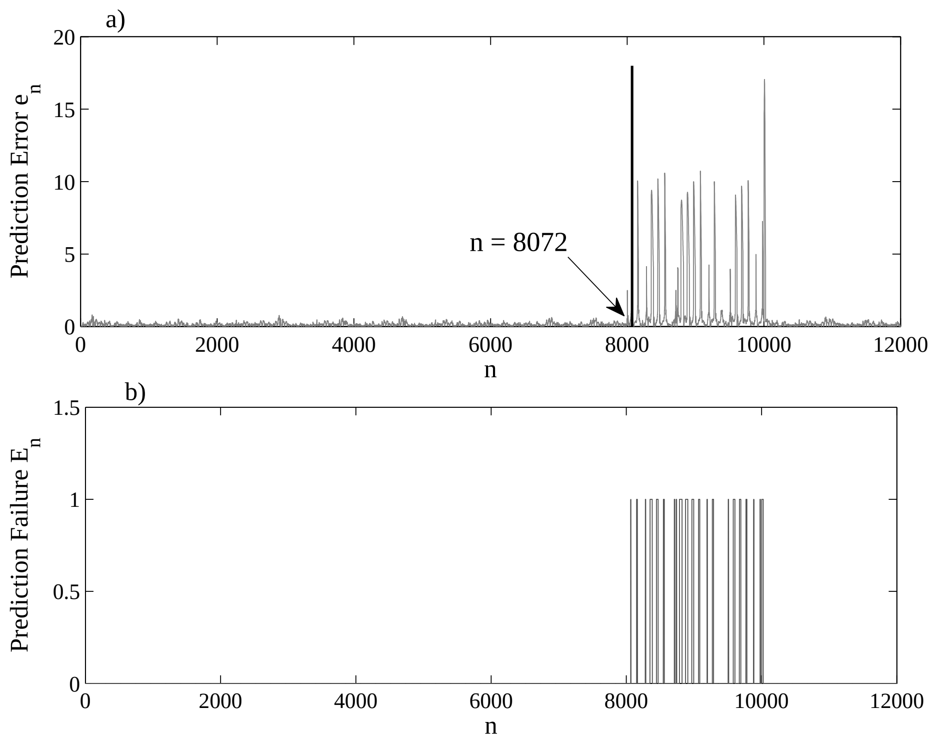


**Fig. 3.** Lorenz system. First component shown in 2 with a change of the parameter $\beta$ in between samples 8000 and 10000. *a)* Prediction error $e_n$. The detection point at $n = 8072$ is marked out. *b)* Prediction failure $E_n$ from the same signal.

In Figure 3 we can observe that the change in the parameter $\beta$ is clearly evidenced by both the increase in the mean of the prediction error $e_n$ and by the prediction failure $E_n$. The detection point was $n = 8072$. The alternating increases of $e_n$ is due to the bad forecasting from those states in the points of the attractor where the change is more noticeable. These points are also manifested in the prediction failure, where $E_n = 1$. In this example, both $e_n$ and $E_n$ provide a clear evidence of a correct detection of the samples where the change have occurred.

### 3.3 Sil'nikov-like Chaos

It is known that many biological systems have a chaotic behavior and pathologic states are often related to changes in the dynamics of the system. Friedrich and Uhl [16] showed that the characteristic behavior of EEG signals with petit-mal epilepsy is related to Sil'nikov type dynamics. Several mathematical models have this sort of behavior. Here we consider the one proposed in [17], defined as:

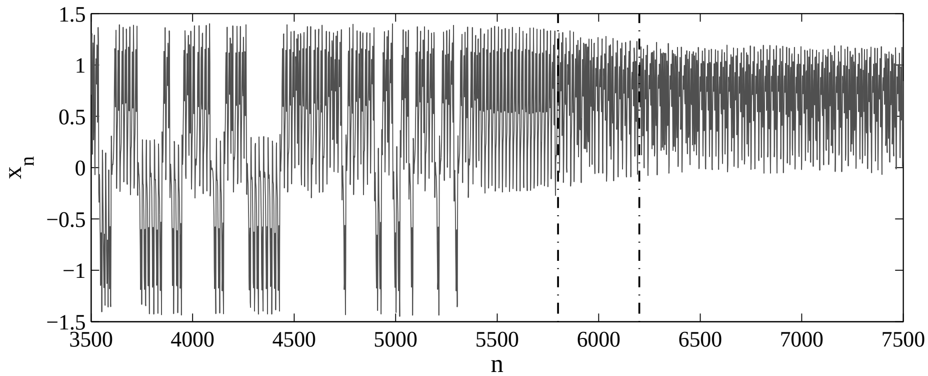


**Fig. 4.** Sil'nikov-like chaos signal. Vertical lines indicate the interval where parameter $a$ in equation (8) changes according to equation (9).

$$\begin{cases} x'(t) = y \\ y'(t) = z \\ z'(t) = \mu x - y - \varepsilon z - a x^2 - b x^3 \end{cases} \tag{8}$$

For certain values of the parameters this system exhibits a chaotic behavior. In particular, for $a \neq 0$, $\mu = 0.65$, $\varepsilon = 0.55$, and $b = 0.65$, the dynamical system in (8) generates the essential behavior of the brain. A chaotic behavior is obtained in the signal $x(t)$ from (8) when fixed values of the parameter $a$ are considered ($a = 0.008$ or $a = 0.2217$). In order to simulate the dynamics of the brain, we evaluate the signal $x(t)$ with a smooth change in the parameter $a$ in the sample interval $[n_c - r\,,\ n_c + r]$, according to:

$$a(n) = \frac{a_1 + a_2}{2} + \frac{a_2 - a_1}{\pi} \tan^{-1}\left(\frac{n - n_c}{r}\right) \tag{9}$$

where $a_1 = 0.008$ and $a_2 = 0.2217$. Seeking to study a gradual change in the dynamics, we set the change radius $r = 200$, centered at the point $n = 6000$. The system was solved using a Runge-Kutta method. The studied signal is shown in Figure 4.

For the analysis, the embedding parameters where chosen to be $m = 3$ and $\tau = 3$. The detection algorithm was run with $\epsilon = r_{min}$, using as a template the first 3000 samples of the signal.

The results of this example are shown in Figure 5. Note that the detection point obtained by the algorithm is $n = 6110$, which is within the changing interval. The increase in the mean prediction error $e_n$ is noticeable, as well as the jump of the prediction failure $E_n$ from zero to one.

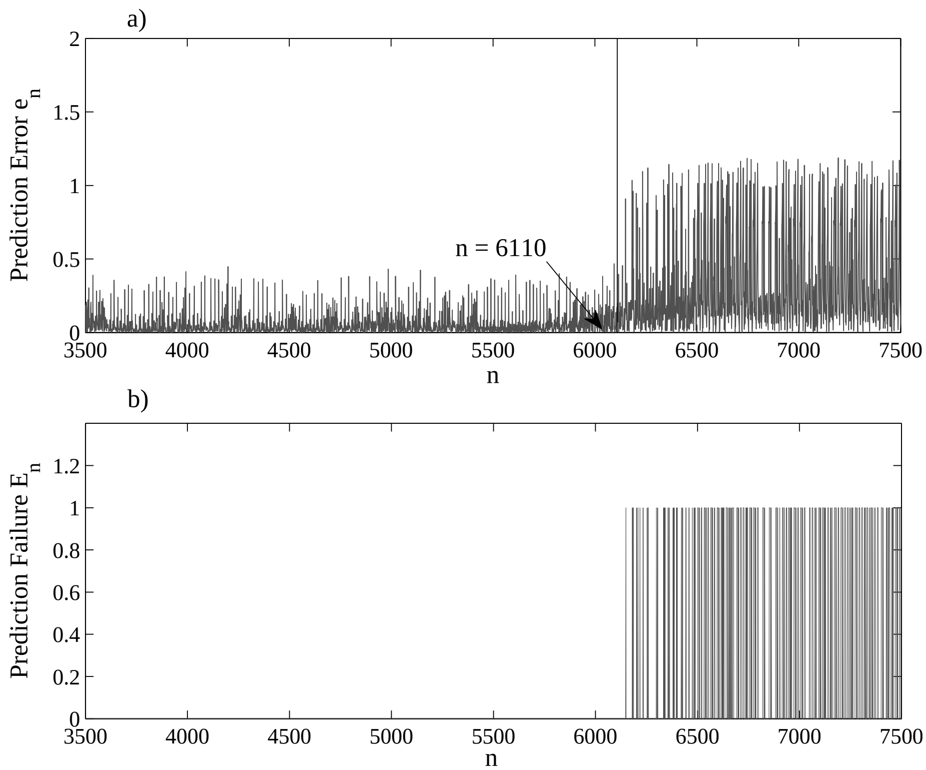


**Fig. 5.** *a)* Prediction error $e_n$ of the signal in (4), where the detection point has been marked out in $n = 6110$. *b)* Prediction failure $E_n$.

## 4 Conclusion

In this paper, we have proposed a new method for the automatic detection of slight changes of the parameters in nonlinear dynamics. It has been applied to deterministic signals of both discrete and continuos systems, and also to a simulated biological model, showing in all cases its ability to accurately detect the changes.

Our method is based on a non-parametric nonlinear prediction algorithm. In order to make it reliable, it is necessary to select a template where the parameters of the governing system are assumed to remain constant. Then, the phase-space is reconstructed by time-delay embedding.

It is known that if some parameter change occurs, the dynamics and the corresponding phase space of the system will change. In this case, the forecast from the template of the signal will be inaccurate. In this work we propose two indices in order to measure this: the absolute *prediction error* ($e_n$) and the binary quantity *prediction failure* ($E_n$). The first one has shown to be a reliable measure of change in the dynamics and it shows fluctuations that are inherent to the proposed method. The addition of CUSUM algorithm for mean change detection allows to automatically detect the changing point. On the other hand, $E_n$ enables us to detect when the change in the system is big enough not to find neighbors in the reconstructed phase space. In future works we will compare these two indices studying their robustness. We will also conduct studies in order to determine optimal values for the parameters $\epsilon$ and $\Delta n$.

We have shown that this nonlinear approach for the detection of parameter changes is promising. Further studies will be conducted to study the performance of the method on real biological signals. Slightly more complex detection algorithms could provide a robust tool for identification of changes in complex nonlinear systems.

## References


1. Päivinen, N., Lammi, S., Pitkänen, A., Nissinen, J., Penttonen, M., Grönfors, T.: Epileptic seizure detection: A nonlinear viewpoint. Computer Methods and Programs in Biomedicine **79** (2005) 151–159
2. Li, X., Ouyang, G.: Nonlinear similarity analysis for epileptic seizures prediction. Nonlinear Analysis: Theory, Methods & Applications **64** (2006) 1666–1678
3. Maiwald, T., Winterhalder, M., R. Aschenbrenner-Scheibeand H. U. Vossand A. Schulze-Bonhageand J. Timmer, J.: Comparison of three nonlinear seizure prediction methods by means of the seizure prediction characteristic. Physica D: Nonlinear Phenomena **194** (2004) 357–368
4. Iasemidis, L.D., Shiau, D.S., Pardalos, P.M., Chaovalitwongse, W., Narayanan, K., Prasad, A., Tsakalis, K., Carney, P.R., Sackellares, J.C.: Long-term prospective on-line real-time seizure prediction. Clinical neurophysiology **116** (2005) 532–544
5. Altunay, S., Telatar, Z., Erogul, O.: Epileptic EEG detection using the linear prediction error energy. Expert Systems with Applications **37** (2010) 5661–5665
6. Sugihara, G., May, R.M.: Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series. Nature **334** (1990) 734–741
7. Dushanova, J., Popivanov, D.: Nonlinear prediction as a tool for tracking the dynamics of single-trial readiness potentials. Journal of Neuroscience Methods **70** (1996) 51–63
8. Torres, M., Gamero, L., Flandrin, P., Abry, P.: On a multiresolution entropy measure. In: Proceedings of SPIE - The International Society for Optical Engineering. Volume 3169. (1997) 400–407
9. Torres, M.E., Añino, M.M., Schlotthauer, G.: Automatic detection of slight parameter changes associated to complex biomedical signals using multiresolution q-entropy. Medical Engineering & Physics **25** (2003) 859–867
10. Añino, M.M., Torres, M.E., Schlotthauer, G.: Slight parameter changes detection in biological models: a multiresolution approach. Physica A: Statistical Mechanics and its Applications **324** (2003) 645–664
11. Schreiber, T.: Interdisciplinary application of nonlinear time series methods. Physics Reports **308** (1999) 1–64
12. Kantz, H., Schreiber, T.: Nonlinear Time Series Analysis. Cambridge University Press, Dresden (2004)
13. Fraser, A.M., Swinney, H.L.: Independent coordinates for strange attractors from mutual information. Physical Review A **33** (1986) 1134–1140

14. Page, E.S.: Continuous inspection schemes. Biometrika **41** (1954) 100–115
15. Nikiforov, I.V., Basseville, M.: Detection of Abrupt Changes : Theory and Application. Prentice-Hall, New Jersey (1993)
16. Friedrich, R., Uhl, C.: Spatio-temporal analysis of human electroencephalograms: Petit-mal epilepsy. Physica D: Nonlinear Phenomena. **98** (1996) 171–182
17. Coullet, P., Tresser, C., Arnéodo, A.: Transition to stochasticity for a class of forced oscillators. Physics Letters A **72** (1979) 268–270