Therefore, if at least one neighbor is found inside the hypersphere of radius ϵ centered at $\mathbf{y}_n$, E_n will be zero. Otherwise, if no neighbors are found in $\mathscr{U}_n$, the algorithm will not be able to perform any forecast and so $E_n = 1$. Thus, the prediction failure is a binary measure of how good the template is to predict the future samples of the given time series x_n.

This suggests that if the predictions are performed with a template taken from an interval where the parameters of the underlying system are assumed to be constant, both the prediction error and the prediction failure would increase their mean values as the dynamics of the system changes. Indeed, a change in some parameter of the system will be reflected on the shape of the reconstructed attractor. So, when searching for neighbors in the trajectory after the change, either the states inside $\mathscr{U}_n$ will cast bad predictions, or no neighbor will be found at all.

In order to perform this detection automatically, we implemented and applied a cumulative sum (CUSUM) algorithm to the absolute error prediction, enabling to obtain a detection point given an appropriate threshold. The CUSUM algorithm was first proposed by Page [14]. The method consists in studying the logarithmic likelihood rate, S_k, which presents a negative slope as long as the mean value doesn't change, and a positive slope after the change. The stopping point is the sample at which the difference between S_k and its minimum value m_k exceeds certain threshold, h_k, which is a free parameter [15].

3 Results and Discussion

In order to test the method described in the previous section, we have applied it to three simulated data with well known chaotic behavior, corresponding to discrete maps and continuos systems. We have imposed slight changes in only one parameter governing their complex dynamics.

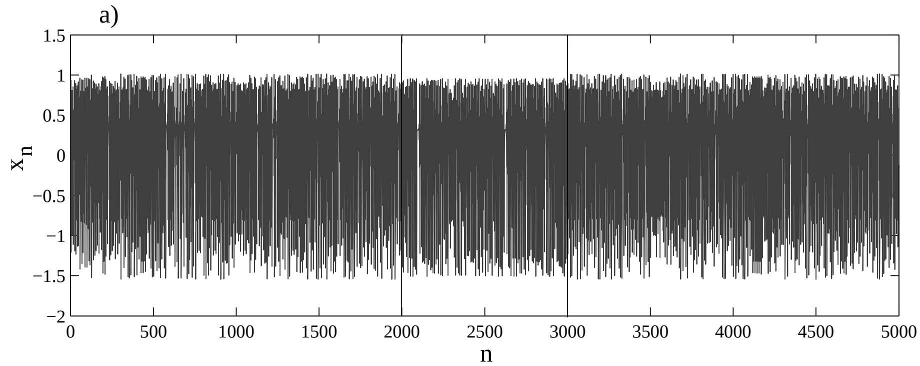
3.1 Henon Map

The Henon map is a discrete-time system given by:

$$\begin{cases} x_{n+1} = 1 - \alpha\, {x_n}^2 + y_n \\ y_{n+1} = \beta\, x_n \end{cases} \tag{6}$$

For certain values of the parameters α and β the system's dynamics is chaotic [12]. In this example, the signal was obtained using the classical values $\alpha = 1.40$ and $\beta = 0.30$, except from $n = 2001$ to $n = 3000$, where $\alpha = 1.32$. Note that there are two abrupt changes in the value of α. The resulting data is shown in Figure 1.a, where the vertical lines indicate where the parameter α changes.

In this example, the embedding parameters are $\tau = 1$ and $m = 2$. We used as prediction parameters $\Delta n = 1$ and $\epsilon = 0.3\, r_{min}$. The template was taken as the first 1000 samples of the signal. The obtained prediction error e_n is shown in Figure 1.b, and the prediction failure index E_n in Figure 1.c. A noticeable increase in the mean value of e_n can be observed in all the interval where the



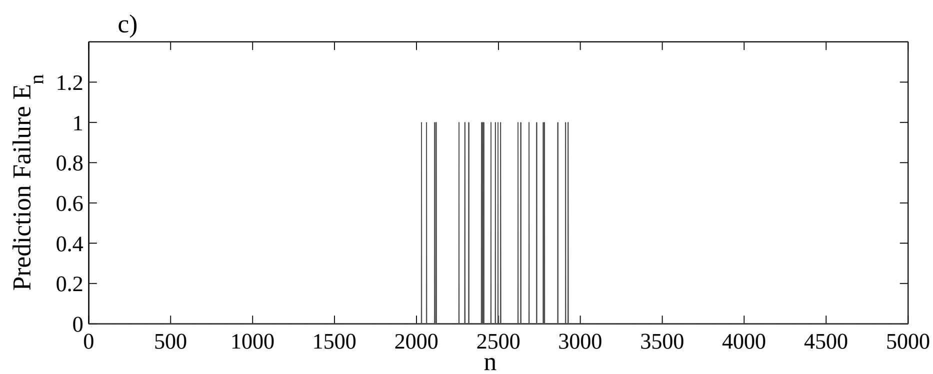


Fig. 1. Henon map. *a)* Henon-map signal studied, where the parameter is changed in the interval 2001–3000. *b)* Prediction error e_n of the signal in (a). The detection point $n = 2011$ has been obtained by the CUSUM analysis. *c)* Prediction failure E_n.

parameter differs from the one in the template. The prediction failure index intermittently goes to one as the change in the dynamics is such that the method can't find any neighbors, providing strong evidence of change in the system.

The value of ϵ accounts for the sensitivity of the algorithm. If the reconstructed phase space of the template (or attractor in the case of chaotic dynamics) is not

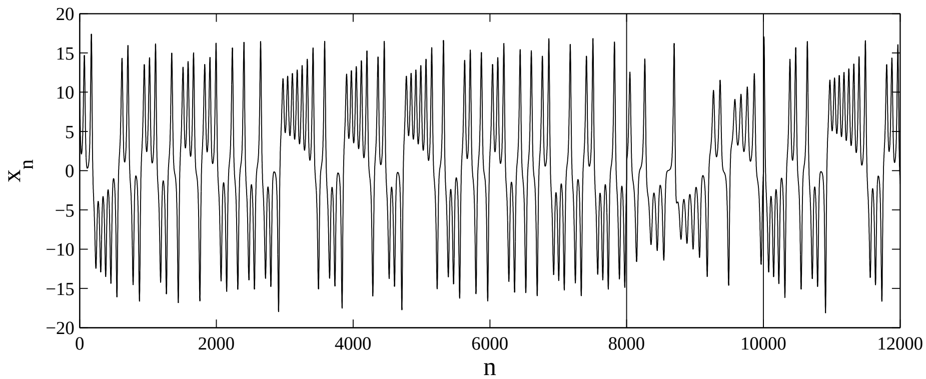


Fig. 2. Signal $x(t)$ corresponding to Lorenz system, with a change of parameter β in between samples 8000 and 10000.

too disperse (i.e. the trajectories don't diverge too quickly) then the radius might be set to a lower value than r_{min} and still have neighbors for every state of the template. Consequently, the algorithm would be more specific and more subtle changes might be detected.

On the other hand, the bigger the template is, the more number of trajectories will be reconstructed in the attractor of the system. Therefore, the distance r_{min} will be lower, which means that smaller radii could be used in the algorithm, making it more specific and sensitive.

3.2 Lorenz System

The Lorenz system is a well know continuos system given by three ordinary differential equations, originally developed as a model for convection between plates. It is given by:

$$\begin{cases} x'(t) = \sigma(y - x) \\ y'(t) = x(\rho - z) - y \\ z'(t) = xy - \beta z \end{cases} \tag{7}$$

Its chaotic behavior for some values of its parameters has became a symbol of chaos theory, and Lorenz system has became one of the most studied ones.

In this example we take as the signal under study the first component $x(t)$ of the system in (7). Classical parameters $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$ have been considered, with a sampling frequency of 100 samples per time unit. The Runge-Kutta method was employed. A 2000 samples segment corresponding to $\beta = 1.5$ was added in between the signal, from sample 8000 to 10000. The resulting signal is shown in Figure 2.

The proposed algorithm was then run using $\epsilon = r_{min}$, with embedding parameters $\tau = 12$ and $m = 3$. The first 3000 samples of the signal have been selected as the template for the phase space reconstruction.

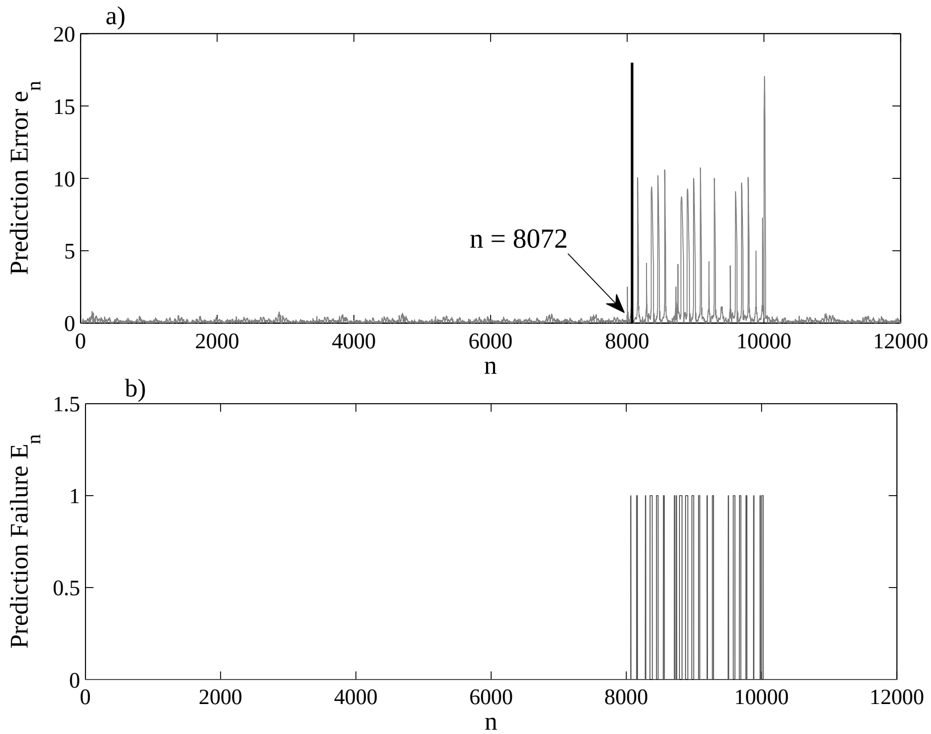


Fig. 3. Lorenz system. First component shown in 2 with a change of the parameter β in between samples 8000 and 10000. *a)* Prediction error e_n. The detection point at $n = 8072$ is marked out. *b)* Prediction failure E_n from the same signal.

In Figure 3 we can observe that the change in the parameter β is clearly evidenced by both the increase in the mean of the prediction error e_n and by the prediction failure E_n. The detection point was $n = 8072$. The alternating increases of e_n is due to the bad forecasting from those states in the points of the attractor where the change is more noticeable. These points are also manifested in the prediction failure, where $E_n = 1$. In this example, both e_n and E_n provide a clear evidence of a correct detection of the samples where the change have occurred.

3.3 Sil'nikov-like Chaos

It is known that many biological systems have a chaotic behavior and pathologic states are often related to changes in the dynamics of the system. Friedrich and Uhl [16] showed that the characteristic behavior of EEG signals with petit-mal epilepsy is related to Sil'nikov type dynamics. Several mathematical models have this sort of behavior. Here we consider the one proposed in [17], defined as:

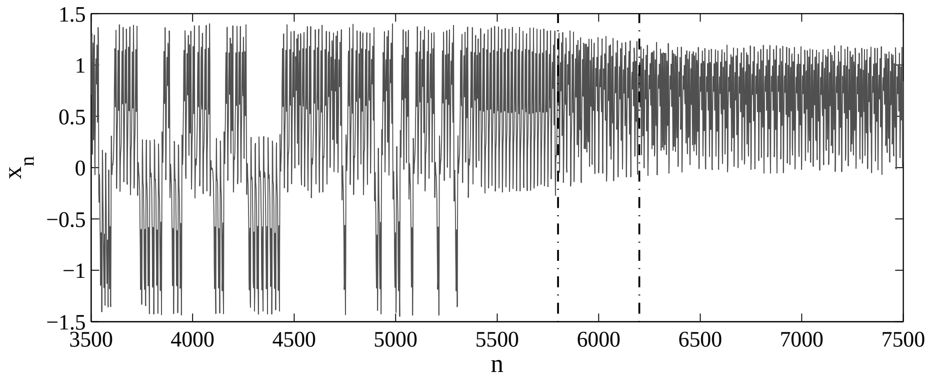


Fig. 4. Sil'nikov-like chaos signal. Vertical lines indicate the interval where parameter a in equation (8) changes according to equation (9).

$$\begin{cases} x'(t) = y \\ y'(t) = z \\ z'(t) = \mu x - y - \varepsilon z - a x^2 - b x^3 \end{cases} \tag{8}$$

For certain values of the parameters this system exhibits a chaotic behavior. In particular, for $a \neq 0$, $\mu = 0.65$, $\varepsilon = 0.55$, and $b = 0.65$, the dynamical system in (8) generates the essential behavior of the brain. A chaotic behavior is obtained in the signal $x(t)$ from (8) when fixed values of the parameter a are considered ($a = 0.008$ or $a = 0.2217$). In order to simulate the dynamics of the brain, we evaluate the signal $x(t)$ with a smooth change in the parameter a in the sample interval $[n_c - r\,,\ n_c + r]$, according to:

$$a(n) = \frac{a_1 + a_2}{2} + \frac{a_2 - a_1}{\pi} \tan^{-1}\left(\frac{n - n_c}{r}\right) \tag{9}$$

where $a_1 = 0.008$ and $a_2 = 0.2217$. Seeking to study a gradual change in the dynamics, we set the change radius $r = 200$, centered at the point $n = 6000$. The system was solved using a Runge-Kutta method. The studied signal is shown in Figure 4.

For the analysis, the embedding parameters where chosen to be $m = 3$ and $\tau = 3$. The detection algorithm was run with $\epsilon = r_{min}$, using as a template the first 3000 samples of the signal.

The results of this example are shown in Figure 5. Note that the detection point obtained by the algorithm is $n = 6110$, which is within the changing interval. The increase in the mean prediction error e_n is noticeable, as well as the jump of the prediction failure E_n from zero to one.

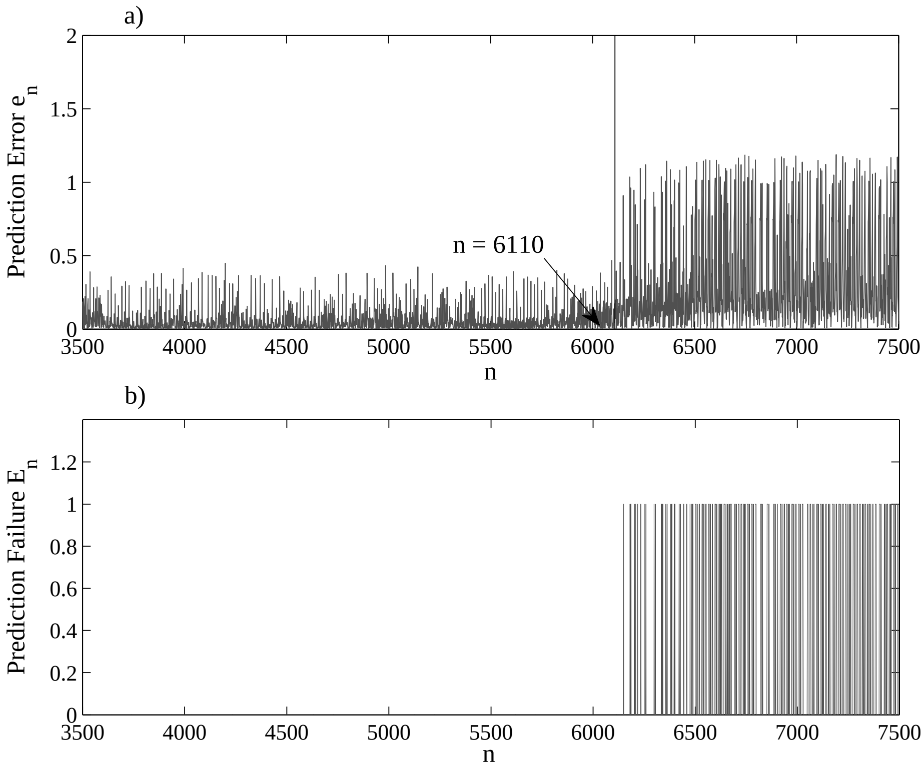


Fig. 5. *a)* Prediction error e_n of the signal in (4), where the detection point has been marked out in $n = 6110$. *b)* Prediction failure E_n.

4 Conclusion

In this paper, we have proposed a new method for the automatic detection of slight changes of the parameters in nonlinear dynamics. It has been applied to deterministic signals of both discrete and continuos systems, and also to a simulated biological model, showing in all cases its ability to accurately detect the changes.

Our method is based on a non-parametric nonlinear prediction algorithm. In order to make it reliable, it is necessary to select a template where the parameters of the governing system are assumed to remain constant. Then, the phase-space is reconstructed by time-delay embedding.

It is known that if some parameter change occurs, the dynamics and the corresponding phase space of the system will change. In this case, the forecast from the template of the signal will be inaccurate. In this work we propose two indices in order to measure this: the absolute *prediction error* (e_n) and the binary quantity *prediction failure* (E_n). The first one has shown to be a reliable measure of change in the dynamics and it shows fluctuations that are inherent to the proposed method. The addition of CUSUM algorithm for mean change detection allows to automatically detect the changing point. On the other hand, E_n enables us to detect when the change in the system is big enough not to find neighbors in the reconstructed phase space. In future works we will compare these two indices studying their robustness. We will also conduct studies in order to determine optimal values for the parameters ϵ and Δn.

We have shown that this nonlinear approach for the detection of parameter changes is promising. Further studies will be conducted to study the performance of the method on real biological signals. Slightly more complex detection algorithms could provide a robust tool for identification of changes in complex nonlinear systems.

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